

Development and Solution of a Multi-Stage Inspection Decision Model Based on Genetic Algorithms

Keyi An^{1,a,*}, Yuxi Pu^{2,b}, Jiayu Zhang^{1,c}, Jiaxin Li^{1,d}

¹ College of Computer and Software, Chengdu Jincheng College, Chengdu, China

² The College of Accounting and Finance, Chengdu Jincheng College, Chengdu, China

^a 1475994406@qq.com, ^b 1831361301@qq.com, ^c 948331996@qq.com, ^d 422803379@qq.com

*Corresponding author

Keywords: Multi-stage decision-making model, Integer programming, Monte Carlo algorithm, Genetic algorithm

Abstract: In this paper, we develop a multi-stage decision-making model aimed at minimizing costs in a production process involving multiple stages and components. The model is formulated using integer programming and solved with a Monte Carlo algorithm. We further enhance the optimization process by employing a genetic algorithm. Additionally, we assume that the defect rates of components, semi-finished, and finished products are determined through random sampling inspection. Using theoretical formulas for sampling estimation, we calculate the upper defect rate limit at 15.9% with 95% confidence and the lower limit at 3.64% with 90% confidence. These defect rates are then used to adjust the model parameters, allowing for a reevaluation of production phase decisions and average production costs.

1. Introduction

With the rapid growth of the market demand for electronic products, enterprises are faced with more complex decision-making problems in the production process, especially in the procurement of spare parts, assembly and quality control. In order to ensure product quality, enterprises not only need to rationally design the testing process but also need to effectively reduce production costs and defective rates. Especially in the quality of finished products, enterprises also need to deal with the dismantling and exchange of substandard products.

Xianli Wu and Huchang Liao [1] introduced the notions of trust radius and alternative chain to model an expert's social influence and to depict multiple alternatives that can be coordinated across adjacent phases. They proposed a multi-phase MCGDM approach based on these concepts. Kezhong Liu et al. [2] developed a multi-phase, multi-objective CADM optimization method, addressing the uncertainty of a ship's path and the crew's risk preferences while meeting safety and economic criteria. Martín Muñoz-Salcedo et al. [3] designed a long-term economic function optimization algorithm using mixed integer-constrained convex programming (MIDCP). This robust algorithm incorporates risk management for intermittent energy sources, techno-economic parameters of chosen technologies, and life cycle analysis (LCA) of various energy systems, including energy storage. Yashar Naderzadeh et al. [4] introduced a novel parallel MCTS algorithm with partial backpropagation for distributed storage, named Parallel Partial Backpropagation MCTS (PPB-MCTS). Lin Shao et al. [5] created an indoor heat sink model to monitor real-time changes in temperature and light using sensors. They applied a genetic algorithm to optimize the placement of heat sinks, considering heat distribution and light intensity across different areas. Simulation experiments assessed the impact of various design options on the indoor environment.

In modern production environments, managing the complexity of multiple stages and numerous components is critical for maintaining efficiency and cost-effectiveness. This study addresses the challenge by developing a multi-stage decision-making model that minimizes production costs. We employ an integer programming approach to structure the model, leveraging Monte Carlo simulations

for initial problem-solving. To further refine the solution, we introduce a genetic algorithm, which offers robust optimization capabilities. The model incorporates defect rates of components, semi-finished, and finished products, obtained through random sampling inspection methods. By recalculating defect rate parameters based on sampling estimation, we enhance decision-making in the production process, focusing on optimizing average production costs.

2. Multi-stage decision optimization

2.1. Cost optimization modeling

The core objective is to provide optimized solutions for decision-making at each stage of the production process of the company, minimizing the total cost of production and maximizing profits. To achieve this goal, the team needs to consider the different cost and revenue factors at each stage. The total cost of a company consists of the cost of spare parts, the cost of assembling semi-finished and finished products, the cost of testing, the cost of handling nonconforming products, and the cost of replacement losses.

To build a cost optimization model, the team can construct an objective function to represent the total cost of the firm in the form of:

$$C_{total} = \sum(C_{purchase} + C_{detection} + C_{assembly} + C_{rework}) \quad (1)$$

Where $C_{purchase}$ is the purchase cost of each spare part, $C_{detection}$ is the cost of testing each spare part, semi-finished and finished product, $C_{assembly}$ is the assembly cost of each semi-finished and finished product, C_{rework} is the cost of dismantling or swapping to deal with non-conforming products.

2.2. Cost optimization model solving and result analysis

Due to, the fluctuation of product quality in manufacturing and uncertainty in the testing process, the results of each production may vary. Here, in order to manage the instability in the production process, the distribution of cost data is obtained by repeating the simulation of different production scenarios. By analyzing these simulated cost data and calculating their average values, the average cost of long-term production can be predicted more accurately.

The model takes into account the rate of defective parts, semi-finished products and finished products that may occur during the production process, and calculates the average production cost for different inspection strategies (full, no or partial inspection). Cost factors include:

- 1) Purchasing cost of spare parts and inspection cost.
- 2) Assembly cost of semi-finished products, inspection cost, and cost of reorganizing or disassembling defective products.
- 3) Testing cost of finished products, cost of exchange of inferior products in the market, etc.

The model mainly uses Monte Carlo simulation to simulate the stochastic nature of the defective rate and the production process under different inspection strategies, which in turn is calculated to get the minimum total production cost. The model is solved using python and Monte Carlo algorithm.

The model is solved using the Monte Carlo algorithm in python:

The average production cost for multiple simulations is in the range of \$103~104. This cost range was derived from multiple Monte Carlo simulations, combining the optimization of the detection strategy and the randomness of the production process.

The average production cost is compared with the all-inspection strategy and the no-inspection strategy to find the optimal combination of inspections. This process will help us to strike a balance between inspection cost, production defect rate and total production cost to determine the optimal decision scheme. In order to get a clearer picture of the decision-making scenario, the average production cost of a full inspection and no inspection is visualized and compared.

It is obtained as shown in Figure 1.

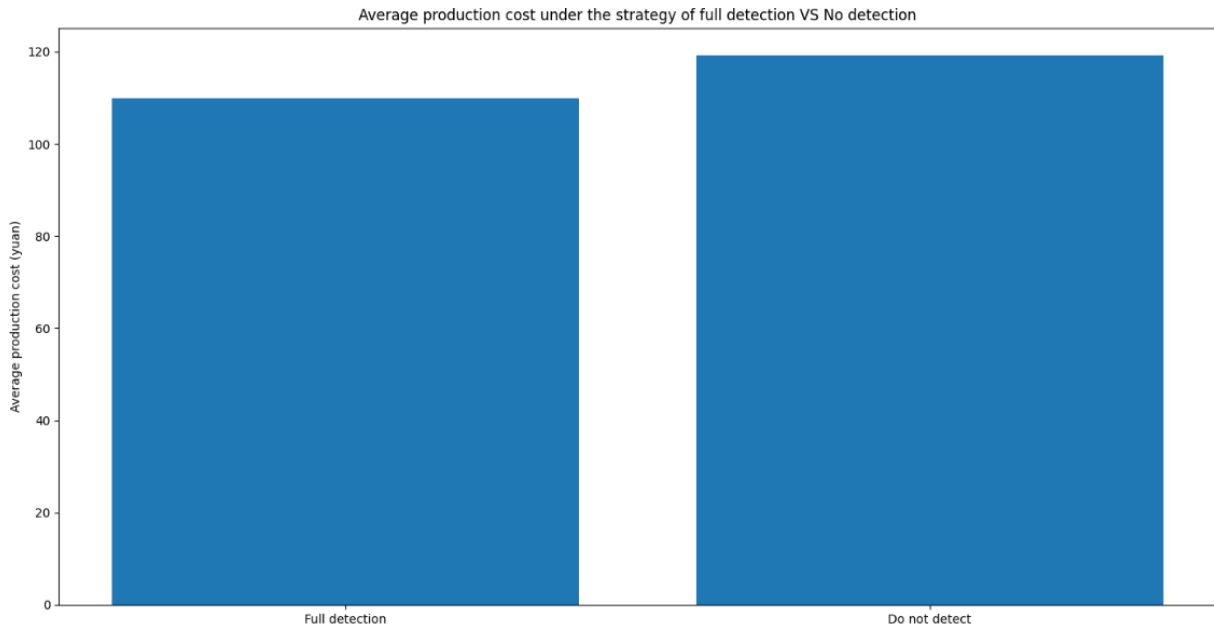


Figure 1 Cost optimization model solving.

The average production cost under the multiple simulation full detection strategy is approximately \$106-\$112. The average production cost under the multiple simulation no-detection strategy is approximately between \$116-120.

The average production cost of the full detection strategy is lower than that of the incomplete detection strategy, which is a relatively rare phenomenon because usually, full detection increases the detection cost, while incomplete detection saves this detection cost. In this case, there may be some defective products that are not detected and get into the finished product or the market due to the fact that when the incomplete inspection strategy is implemented. These undetected defective products may need to be disassembled or replaced at a later stage, which will bring a higher cost of \$40 per swap. In contrast, although the full inspection strategy requires higher inspection costs at the early stage of production, which may range from \$1 to \$2 per inspection, it can effectively avoid the high dismantling and replacement costs due to defective products at a later stage. Therefore

In the long run, the full-inspection strategy helps to reduce the overall production cost because it reduces the possible larger losses in the later stages by investing in the early stages.

To summarize:

All-inspection strategy: although it ensures quality, the cost of inspection is high and the total cost may be unsatisfactory.

No-inspection strategy: high rate of defective products, although it saves on inspection costs, it may increase the subsequent processing costs, which ultimately leads to a higher total cost.

Average production cost: can help us find the balance between the inspection cost and the cost of defective products processing, and then get the cost minimization.

2.3. Introduction of Genetic Algorithms

Monte Carlo simulation techniques are applied to analyze the cost impact of various inspection methods in existing production cost assessment models. To achieve further cost reduction, an optimization algorithm can be considered to automatically determine the optimal combination of inspection strategies.

Here, a classical Genetic Algorithm (GA) is used, which is an optimization algorithm based on the principle of biological evolution that continuously improves a set of solutions by simulating the processes of selection, crossover, and mutation to gradually approximate the optimal solution. The optimal combination of detection strategies is found through the evolutionary process with the goal of minimizing the production cost.

Through a genetic algorithm, the optimal detection strategy can be found automatically and the

optimal production cost corresponding to this strategy can be output.

Detection decisions in production are progressively optimized by a genetic algorithm implemented in Python. In each generation, the genetic algorithm selects, crossovers, and mutates detection strategies to progressively improve these decisions. After several generations of evolution, the algorithm finally finds a set of optimal inspection strategies that minimize the production cost. This strategy not only optimizes the combination of inspection of spare parts, semi-finished products and finished products, but also outputs the lowest production cost.

Obtained:

Optimal Inspection Strategy: [True, False, True, True, False, True, False, True, True, True, True, False]

The details are spare parts 1: detection, spare parts 2: no detection, spare parts 3: detection, spare parts 4: detection, spare parts 5: no detection, spare parts 6: detection, spare parts 7: no detection, spare parts 8: detection, semi-finished products: detection, finished products: detection, finished products without disassembly.

Optimal production cost: \$76.25

The final strategy is the optimal inspection solution after genetic algorithm optimization. It considers how to minimize the production cost by reducing the defective rate while minimizing unnecessary inspection and disassembly work. This is the lowest production cost obtained through the optimized inspection strategy. This cost has been optimized by a multi-generation genetic algorithm to balance the inspection cost and the additional loss due to the defective rate.

Compared to other inspection strategies, this optimal strategy may reduce the inspection cost by reducing unnecessary inspections, while at the same time avoiding excessive substandard products from entering the subsequent production process, keeping the substandard rate low, and reducing the losses from reorganization, disassembly, and swapping.

3. Multi-stage modeling and solving for random sample testing

3.1. Preparation of the model

The theoretical formula for sampling to estimate the rate of defective products can be derived from the statistical theory of binomial distribution. Since the binomial distribution can be approximated as a normal distribution by the Central Limit Theorem, the formula for the sampling estimate of the defective rate usually incorporates the concept of normal distribution.

STEP1: Estimation of defective rate

If we randomly select n samples from a batch of products and find X defective products in these samples, we can define the defective rate of the sample, p to denote the proportion of defective products in the sample:

$$\hat{p} = \frac{X}{n} \quad (2)$$

STRP2: The confidence interval of the defective rate

Based on the central limit theorem, when the sample size n is large enough, the distribution of the sample defective rate p can be approximated as a normal distribution with standard deviation SE denoted as:

$$SE = \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} \quad (3)$$

STEP3: Confidence interval formula

The upper and lower limits of the confidence interval can be calculated using the following formula:

$$\hat{p} \pm Z_{\frac{\alpha}{2}} \times SE \quad (4)$$

To wit:

$$\hat{p} \pm Z_{\frac{\alpha}{2}} \times \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} \quad (5)$$

The upper limit is $\hat{p} + Z_{\alpha/2} \times SE$, and the lower limit is $\hat{p} - Z_{\alpha/2} \times SE$.

Estimates the overall defective rate from sampling and provides a basis for acceptance or rejection decisions in real-world problems.

3.2. Significant difference modeling

For the reject scenario, the sample size is $n=98$, the confidence level is 95%, and the nominal defect rate is 10%.

For the acceptance program, the sample size $n = 60$, the confidence level of 90%, the nominal defective rate of 10%.

1) Reject the program (95% confidence level defective rate over the nominal value) according to the formula:

$$\hat{p} \pm Z_{\frac{\alpha}{2}} \times \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} \quad (6)$$

where $Z_{\frac{\alpha}{2}} = 1.96$ (for 95% confidence level), $n = 98$, and nominal defective rate $p_0 = 0.10$.

The standard error is:

$$SE = \sqrt{\frac{0.10(1 - 0.10)}{98}} = \sqrt{\frac{0.10 \times 0.90}{98}} \approx 0.0304 \quad (7)$$

The confidence interval is:

$$0.10 \pm 1.96 \times 0.0304 = 0.10 \pm 0.059 \quad (8)$$

The substandard rate is capped under the reject program:

$$0.10 + 0.059 = 0.159 \quad (9)$$

2) Reception scenario (90% confidence that the defective rate does not exceed the nominal value)

For the reception scenario, the confidence level is 90%, corresponding to $Z_{\alpha/2} = 1.645$, sample size $n = 60$.

The standard error is:

$$SE = \sqrt{\frac{0.10(1 - 0.10)}{60}} = \sqrt{\frac{0.10 \times 0.90}{60}} \approx 0.0387 \quad (10)$$

The confidence interval is:

$$0.10 \pm 1.645 \times 0.0387 = 0.10 \pm 0.0636 \quad (11)$$

The substandard rate is capped under the receiving program:

$$0.10 - 0.0636 = 0.0364 \quad (12)$$

Then it is obtained:

In the reject scenario (95% confidence that the defective rate exceeds the nominal value), the parts can be rejected when the defective rate exceeds 15.9%.

In the acceptance program (90% confidence that the defective rate does not exceed the nominal value), the parts can be accepted if the defective rate is less than 3.64%.

If the defect rate is less than 3.64%, the parts can be accepted.

3.3. Solving the model and analyzing the results

When the defective rate fluctuates upwards: choose a more cautious inspection strategy, including inspecting more spare parts and finished products, to minimize the risk of defective products entering the market. Increase the number of operations that dismantle non-conforming finished products to ensure that they do not reach the market, even though this increases costs. When the defective rate fluctuates downwards: you can reduce the inspection operation, especially for spare parts and finished products, thus reducing the inspection cost. Reduce unnecessary dismantling operations, as the number of non-conforming finished products will also be reduced when the defective rate is lower.

The impact of fluctuating defect rates on total costs is significant. Upward fluctuations increase inspection and disassembly costs, while downward fluctuations reduce these operations and lower total costs. Flexibility in decision making: Companies can choose different inspection and disassembly strategies to optimize cost control in the production process, depending on the variation in the defective rate.

When making decisions about a company's production process involving multiple processes and multiple spare parts. Based on the 15.9% and 3.64% defective rate upper and lower limits obtained from sample testing, redefine the defective rate component and adjust the defective rate thresholds to make inspection, assembly and disassembly decisions. The threshold for defective rate is adjusted to make inspection, assembly, and disassembly decisions. 5.9% is floated upward from 10% and 6.64% is floated downward from 10%. To optimize the inspection, assembly and disassembly strategies in the production process.

The upward and downward cost adjustments are shown in Figure 2 and Figure 3.

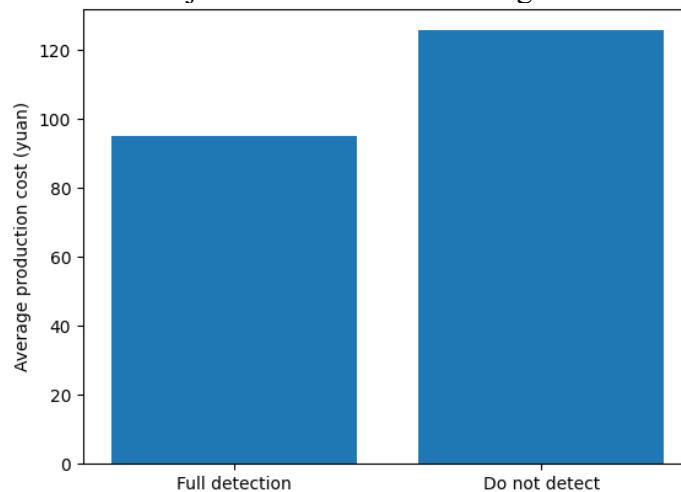


Figure 2 Raise costs.

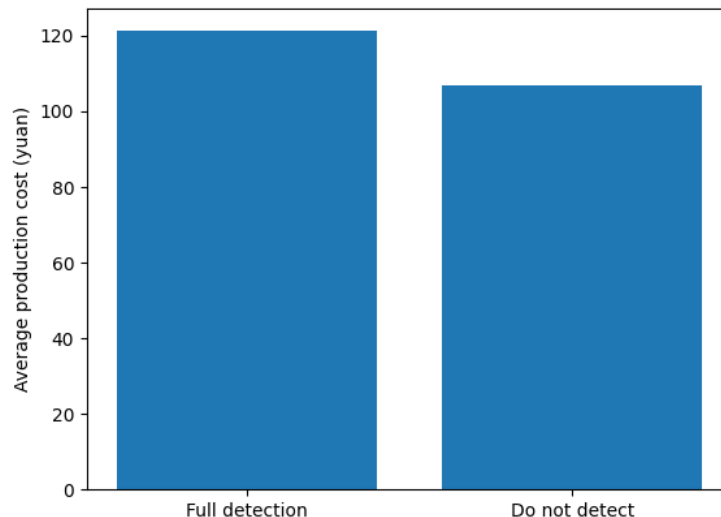


Figure 3 Reduce costs.

After a 5.9% upward fluctuation in the defective rate:

The average production cost is approximately \$90.

Average production cost with full inspection strategy: \$98.00.

Average production cost with no inspection strategy: \$125.64.

After a downward fluctuation of 6.64% in the defect rate:

The average production cost is approximately \$117.

Average production cost with full inspection strategy: \$122.45.

Average production cost with no inspection strategy: \$106.96.

When the defective rate is high, the all-inspection strategy can reduce the total production cost by preventing defective products from entering the market, although there is an initial inspection cost. The no-inspection strategy eliminates the initial inspection cost, but with a higher defective rate, the total cost increases instead, due to the much higher follow-up and rework costs associated with defective products coming into the market. By comparing the average cost after fluctuating the defective rate upwards, it can be seen that when the defective rate is high, the all-inspection strategy as far as possible is still an effective means to ensure the quality of the finished product.

When defect rates are low, a no-inspection strategy is more cost effective. Because of the high cost of testing, the testing expense becomes the main cost source for the company compared to the risk of defective finished products brought by no testing. By comparing the average cost after downward fluctuation of the defective rate, it can be seen that the partial inspection strategy can guarantee the quality of finished products when the defective rate is low.

Therefore, the full inspection strategy has more cost advantages when the defective rate rises, while the inspection can be reduced moderately according to the actual situation when the defective rate falls, but still should maintain a certain degree of inspection to control the risk. Enterprises can flexibly adjust the inspection strategy according to the current defective rate, balancing quality control and cost optimization.

4. Conclusion

This research presents a comprehensive approach to optimizing multi-stage production processes through a cost-minimization model. By integrating integer programming with Monte Carlo and genetic algorithms, we provide a robust framework for decision-making in complex production settings. The incorporation of defect rate estimation through sampling enhances the model's accuracy and reliability. Our findings demonstrate the potential for significant cost reductions and improved production efficiency. Future work could explore further refinement of the genetic algorithm and its application to other complex systems.

References

- [1] Wu, X. and Liao, H., (2024) A multi-stage multi-criterion group decision-making method for emergency management based on alternative chain and trust radius of experts. *International Journal of Disaster Risk Reduction*, 101, 104253.
- [2] Liu, K., Wu, X., Zhou, Y., Yuan, Z., Xin, X. and Zhang, J., (2023) Coordinated multi-stage and multi-objective optimization approach for ship collision avoidance decision-making. *Ocean Engineering*, 287, 115888.
- [3] Muñoz-Salcedo, M., Ruiz de Adana, M. and Peci-López, F., (2024) Mixed-integer disciplined convex programming approach applied to the optimal energy supply of near-zero energy buildings. *Heliyon*, 10(17), e36873.
- [4] Naderzadeh, Y., Grosu, D. and Chinnam, R. B., (2024) PPB-MCTS: A novel distributed-memory parallel partial-backpropagation Monte Carlo tree search algorithm. *Journal of Parallel and Distributed Computing*, 193, 104944.
- [5] Shao, L., Li, A. and Wang, P., (2024) Simulation of indoor thermal energy radiators and visualization design of indoor environment layout based on genetic algorithm and light sensor. *Thermal Science and Engineering Progress*, 55, 102903.